Fuzzy Production Inventory Model with Stock Dependent Demand Using Genetic Algorithm (GA) Under Inflationary Environment

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ABSTRACT

In this article, we want to solve the complication of production of the product for the newly launched product and integrate it with the value of time, product, and inflation value. To address such problems, we have used a linear demand rate, which is directly proportional to the stock level i.e. if the stock level is maximum, the demand will automatically increase and the inventory level will also increase. If the level of the stock decreases then the demand and inventory level will also decrease. Moreover, the production will be stopped, when the level of stock will reach level S and there is no effect of demand. The S_0 stock level is definite. The prospective of this research is to increase the total profit of the model. For which the use of the Centroid method of defuzzification is used to defuzzify the total profit. This model will be explained with the help of numerical examples and sensitivity analysis and Java and MATLAB R2015a are used to get the optimal values of this model.

Keywords: Centroid Method, EPQ model, genetic algorithm, stock-dependent demand, Supply Chain Management

INTRODUCTION

India is a country of diversity where within 50 km people is speaking different languages and they are having different requirements during time to time and it changes day by day. This kind of situation is called as the demand-reliant on the stock. In this field, so many mathematical models have already been discussed in the prevailing literature. One of these models was by Chang et al. (2004), Datta et al. (1998), Mishra (1979),

Singh et al. (2007), Singh et al. (2010), Teng and Chang (1995), and Yao et al. (2000). These conventional inventory models are installed on the root where all the obstacles are fixed in the total cost and they have some parameters or value. But the time and place are different for the place of market setup and the value of the parameters or constraints may increase or decrease according to the environment or the needs of the people. To handle these types of situations, we will assume that all the criteria are fuzzy and the resulting function will also be fuzzy.

To get an impression about this research, we refer to Chandra and Bahner (1985), Maity and Maiti (2007), Teng and Chang (2005), Singh and Singh (2010), and Wu et al. (2006). Classical inventory models neglect the effects of inflation because inflation does not affect inventory rules by any significant degree but we cannot ignore inflation because it is directly or indirectly proportional to return on investment.

Inflation should be especially measured for long period investment and forecast such as ice and cold storage industry and weather forecasts, which specifically help in the production industry (Agarwal et al., 2017). Discussing in which to deal with the actual situation of the ice and cold storage (Agra) and to find the solution to the problem. To get the solution of these problems they used GA.

To get an impression on this research, we refer to Balkhi and Benkherouf (2004), Buzacott (1975), Lieo et al. (2000), Mehta and Shah (2003), Yao and Lee (1999), and Singh and Singh (2010) who discussed GA's experiments in the complex decision-making. In addition, Kumar et al. (2016) and Sarkar et al. (1997), discussed about improving the quality of GA for single/multi-purpose continuous/discrete optimization problems during the last days. They also discussed a model with constant demand which is based on the stock level under inflation environment and GA was the solution of this kind of problems. Michalewich (1992) extends a GA, which is called Contractive Mapping Genetic Algorithm (CMGA).

In this model, they discussed that the new population would be selected when it was proven that the new population or generation was fitter compared to the old population and it had been obtained by the asymptotic convergence of the algorithm by the Banach Fixed Point Theorem.

Bessaou and Siarry (2001) discussed a GA where they were creating more than one solution for the population of solutions in the beginning, then genetic operations were done on every population. Last and Eval (2005) discussed about a model of a GA, which was based on different chrome shapes and different ages (such as child age, young age, middle age and old age) chrome. Pezzella Morgantia and Ciaschettib (2008) extended a GA that assimilated new ones with different sets to re-develop and also satisfied the job-shop scheduling problem.

Comparison is presented in the Table 1.

Table 1
Comparison is presented in the following table of the literature

S. No	Author's	EPQ Model	Inflation	Centroid method	Stock Dependent Demand	GA
1	Agarwal (2017)					Yes
2	Balkhi (2004)				Yes	
3	Bessaou and Siarry (2001).					Yes
4	Buzacott (1975)		Yes			
5	Chandra and Bahner (1985).		Yes			
6	Chang, Yao and Quyang (2004).			Yes		
7	Datta, Paul and Pal (1998).				Yes	
8	Kumar, Agarwal and Kumar (2016).		Yes		Yes	Yes
9	Last and Eyal (2005)			Yes		Yes
10	Liao, Tsai and Su (2000)		Yes			
11	Maity and Maiti (2007).					Yes
12	Mehta and Shah (2003)		Yes			
13	Michalewicz (1992)					Yes
14	Pezzella, Morgantia and Ciaschettib (2008)					
15	Mishra (1979)		Yes			
16	Sarkar, Mukherjee and Balan (1997)		Yes		Yes	
17	Singh, Singh and Singh (2007)	Yes	Yes		Yes	
18	Singh, Kumari and Kumar (2010)	Yes		yes	Yes	
19	Singh and Singh (2010)	Yes	Yes			
20	Singh and Singh (2010)	Yes				
21	Teng and Chang (1995)		Yes		Yes	
22	Teng and Chang (2005)		Yes		Yes	
23	Wu, Ouyang and Yang (2006)				Yes	
24	Yao and Lee (1999)				Yes	
25	Yao, Chang and Su (2000)				Yes	

This paper presents two situations A: $S \leq S_0$ and B: $S > S_0$, where S_0 is the fixed stock level and S is the stock-level when production is terminated. Its major objective is to maximize the total profit on different situations and the environment, along with linear demand rate which is directly related to stock - level, if demand will increase or decrease then the stock level will also increase or decrease. Moreover, Centroid method is used to defuzzify the total profit and we can use GA to improve the result.

Genetic Algorithm (GA)

Genetic Algorithm is the method of computer engineering and operational research which is derived from Charles Darwin theory of biology. In GA, a population is modified until we will not get the optimized solution of our problem. GA uses four major functions to find the optimal solution which are:

- (I)   Fitness: It is determined by an objective or subjective function.
- (II)   Selection: After getting fitness population we select better population to perform further operations.
- (III) Crossover: This is a function of combining two parents for next generation.
- (IV) Mutation**:** It is a function of random variations in individual parents.

Where C_p is the probability of crossover, M_p is the probability of mutation, I_M is an iteration counter in each generation, I_{M0} is the upper limit of iteration counter I_M and I_N is the population size. L is the generation counter, Po (L) is the population of possible solutions for the generation L.

GA Algorithm:

1. Start and
2. Initializing the integer variables L, Po (L), I_M, Po_1 (L), C_p, M_p, I_{M0}, and I_N.
3. Set L = 0 and I_M = 0.
4. Evaluate the variable Po (L).
5. While ($I_M < I_{M0}$) otherwise go to step 16.
6. Select I_N solutions from Po (L) for mating pool using Roulette -Wheel process.
7. Select the solutions from Po (L), for crossover depending on C_p.
8. Now perform crossover operations on the selected solutions.
9. Select the solutions from Po (L), for mutation depending on M_p.
10. Perform mutation operation on selected solutions for getting the population Po_1 (L).
11. Classify the value of Po_1 (L).
12. Set $I_M = I_M$ +1.
13. If the average fitness of Po_1 (L) is greater than average fitness of Po (L) then go to steps 14,15,16. Otherwise, go to step 5
14. Set Po (L +1) = Po_1 (L).
15. Increment the value of L by L+1.
16. Set I_M = 0.
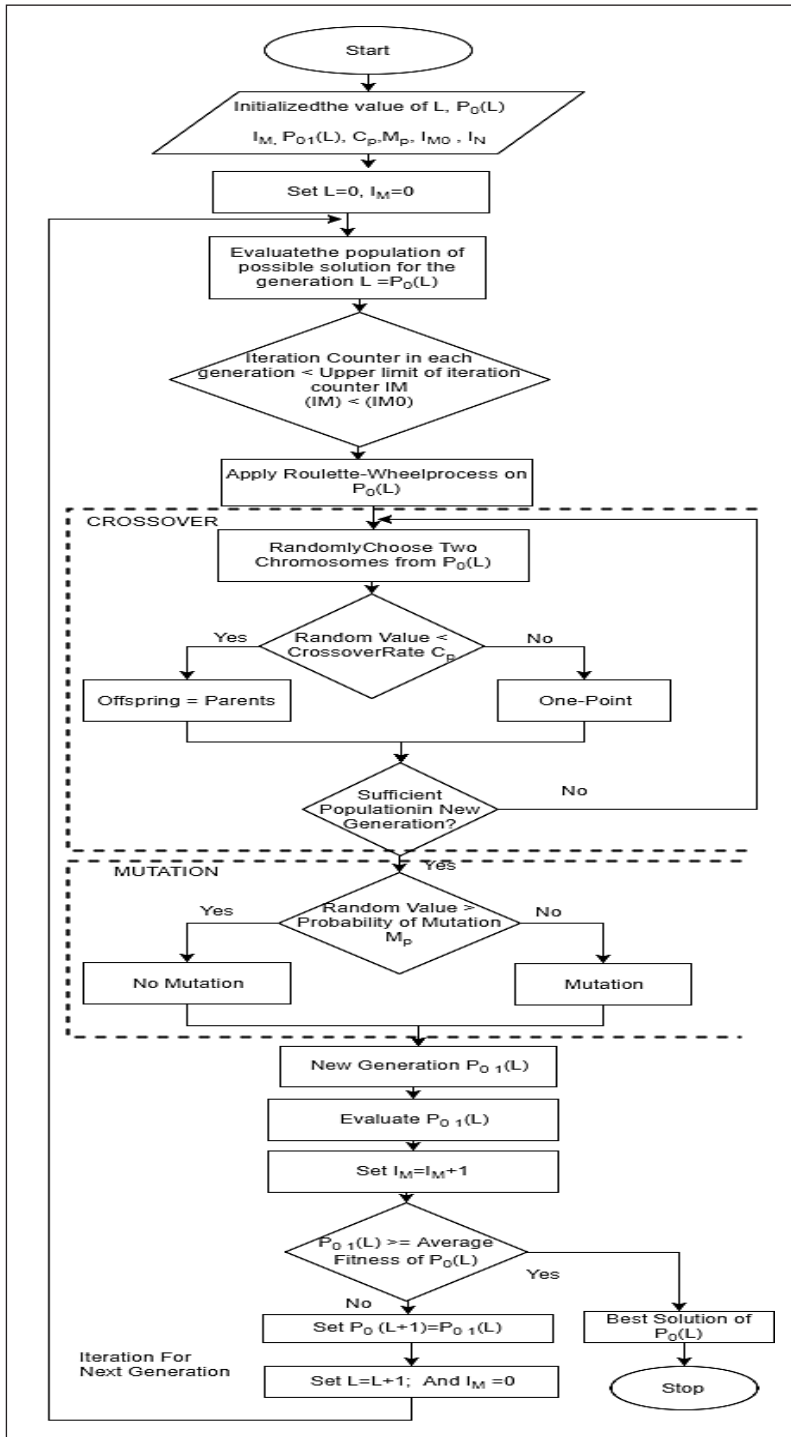17. Print the Best solution value of Po (L).

GA Flow Chart:

Figure 1. Genetic algorithm flow chart

Concept of Triangular Fuzzy Number

Let A= (k_1, k_2, k_3) is a triangular fuzzy number, where $k_1 = k- \Delta_1$, $k_2 = k$ and $k_3 = k + \Delta_2$. Such that $0 < \Delta_1 < k$, $0 < \Delta_2$ and Δ_1, Δ_2 are determined by the result producer depended on the uncertainty of the problem. And then A can be represented as $A = [A_l^L(\alpha), A_l^U(\alpha)] \cup [A_r^L(\alpha), A_r^U(\alpha)]$ subject to the constraint $0 < k_1 < k_2 < k_3$. And the membership function of the triangular fuzzy numbers is defined as follows. $\mu_A(x) : R \to [0,1]$

$$\mu_A(x) = \begin{cases} 0, x < k_1, x > k_3 \\ L(x) = \dfrac{x - k_1}{k_2 - k_1}, k_1 \le x \le k_2 \\ R(x) = \dfrac{k_3 - x}{k_3 - k_2}, k_2 \le x \le k_3 \end{cases}$$
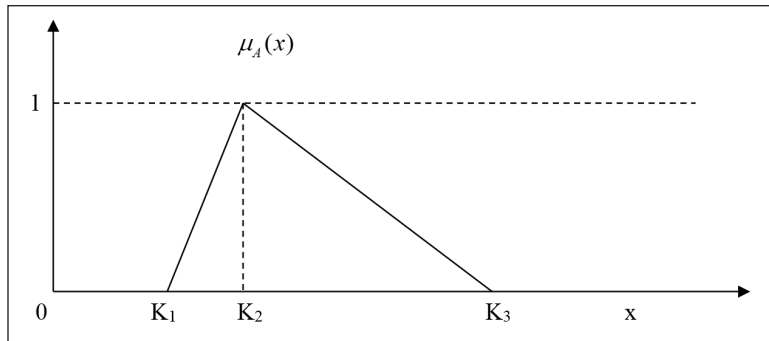

Figure 2. Graphical representation of membership function

Centroid Method

This method is also called the center of gravity or the center of the center. This is the most influential and appeals to all the defuzzification methods given by the arithmetic expression:

$$M_A = \frac{\int \mu_A(x) * x dx}{\int \mu_A(x) dx}$$

Where $\int$ denotes an algebraic integration. Hence the centroid for A is given as

$$M_k = \frac{K_1 + K_2 + K_3}{3} = \frac{k - \Delta_1 + k + k + \Delta_2}{3} = k + \frac{(\Delta_2 - \Delta_1)}{3}$$

Assumptions and Notations

The mathematical model of this study has been developed based on the following assumptions and notations.

Notations. We are using following notations for this model

$\widetilde{c_1}$: Fuzzy holding cost of the inventory item, Rs./ per unit / per unit time.

$\widetilde{c_2}$: Fuzzy deterioration cost, Rs. / per unit.

$\widetilde{c_3}$: Fuzzy ordering cost, Rs. / per order.

$\widetilde{c_{1r}}$: Fuzzy holding cost for raw material inventory, Rs. / per unit / per unit.

$\widetilde{c_{2r}}$: Fuzzy deterioration cost for raw material inventory, Rs. / per unit.

$\widetilde{c_{3r}}$: Fuzzy ordering cost for raw material inventory, Rs. / per order.

$\widetilde{c_p}$: Fuzzy purchasing cost for raw material inventory, Rs./ per unit.

$\widetilde{p_c}$: Fuzzy production cost, Rs. / per unit.

$\widetilde{s_t}$: Fuzzy selling price, Rs. / per unit.

γ : Deterioration Rate.

I: Inflation Rate.
D: Discount Rate.
R: D-I
S: Maximum inventory level of the production cycle.
S_0 : Constant inventory level.
L: Length of the ordering cycle.
I (T): Inventory Level at time T $\in$ [0, L].

Assumptions. The following assumptions, we are using in this model are:
1. There is only one item in the inventory system and the plan horizon is infinite.
2. The Production rate is demand dependent, P(T) = z D(T), where z is a non-negative constant.
3. The Demand rate is D(T) deterministic and given by its functional form

$$D(T) = \begin{cases} \alpha + \beta I(T), \ S > S_0 \\ D, 0 \leq S \leq S_0 \end{cases}$$

   Where $\alpha < 0$, $0 < \beta < 1$, D = c + b (T), and α, β, c, and d are known as parameters of scale and size, respectively.
4. Shortages are not permissible here.

Model Formulation

When the stock level reaches level S, S_0 has stopped production at constant stock-level. Then, in the following two cases, can arise

A: $S \leq S_0$
B: $S > S_0$

Case A: $S \leq S_0$. This is the standard Economic Production Quantity (EPQ) model for deteriorating goods with linear demand rate. At the time $T = 0$ output is started, with zero list level and at that time $T = L_1$ is stopped when the inventory level reaches level S. Due to the combined effect of the demand of that inventory level decreases and the fall in L, on which the inventory level reaches zero level. At any point of time the inventory level can be described by the following differential equation and graphically depicted in the figure-3.
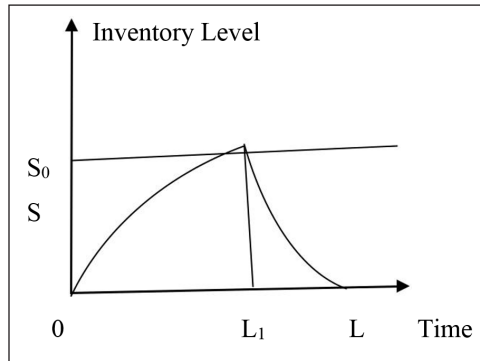

Figure 3. Graphical representation of case A: $(S \leq S_0)$

$$I_1'(T) + \gamma I_1(T) = (z-1)D, \ 0 \leq T \leq L_1 \tag{1}$$

$$I_2'(T) + \gamma I_2(T) = -D, \ L_1 \leq T \leq L \tag{2}$$

The boundary equations are $I_1(0) = 0$ and $I_2(L) = 0$. Solutions of the equation (1) and (2) are:

$$I_1(T) = \frac{(z-1)D}{\gamma} - \frac{(z-1)D}{\gamma} e^{(-\gamma T)} = \frac{(z-1)D}{\gamma}[1 - e^{(-\gamma T)}], \ 0 \leq T \leq L_1 \tag{3}$$

$$I_2(T) = \frac{De^{(\gamma L)}}{\gamma e^{(\gamma t)}} - \frac{D}{\gamma} = \frac{D}{\gamma}\left[e^{\gamma(L-T)} - 1\right], \ L_1 \leq T \leq L \tag{4}$$

From the equations of (3) and (4), Put $I_1(L_1) = I_2(L_1)$, then

$$e^{\gamma L} = z e^{\gamma L_1} - a + 1$$

$$\gamma L = \log[z(e^{\gamma L_1} - 1) + 1]$$

$$L = \frac{1}{\gamma} \log[z(e^{\gamma L_1} - 1) + 1] \tag{5}$$

The Maximum inventory level is $\quad S = I_1(L_1) = \dfrac{(z-1)D}{\gamma}[1 - e^{(-\gamma L_1)}] \tag{6}$

The present value of the fuzzy holding cost of organized inventory is

$$\widetilde{H o C} = \frac{\widetilde{h_2}D}{\gamma^2}\left\{(z-1)e^{(-\gamma L_1)}[(\gamma L_1 - 1)e^{\gamma L_1} + 1] + e^{(-\gamma L_1)}(e^{\gamma L} + \gamma L_1 e^{\gamma L_1}) - \gamma L + 1\right\} \tag{7}$$

The Present worth of fuzzy production cost is $\quad \widetilde{P o C} = \dfrac{\widetilde{c_a}D}{R}(1 - e^{(-RL_1)}) \tag{8}$

The Present worth of fuzzy deteriorating cost is

$$\widetilde{D o C} = \frac{\widetilde{c_2}D}{\gamma}\left\{(z-1)e^{(-\gamma L_1)}[(\gamma L_1 - 1)e^{\gamma L_1} + 1] + e^{(-\gamma L_1)}(e^{\gamma L} + \gamma L_1 e^{\gamma L_1}) - \gamma L + 1\right\} \tag{9}$$

The Present worth of fuzzy sales revenue is $\quad \widetilde{S a R} = \dfrac{\widetilde{s}D}{R}[1 - e^{(-RL)}] \tag{10}$

The Present worth of fuzzy ordering cost is $\quad \widetilde{O o C} = \widetilde{c_3} \tag{11}$

Raw Material Inventory Model during the Time $(0, L_1)$

Initially, the seller buys raw materials in many and produces finished goods. The producer starts production on time $T = 0$, and due to the combined effect of the production and decline of raw materials, $T = L_1$ reaches zero level in time. At any time, the seller's inventory system of raw material can be represented by the following differential equation and shown in the figure 4.
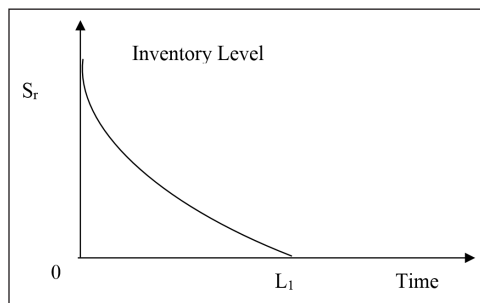

Figure 4. Graphical representation of raw material inventory

$$I_R'(T) + \gamma I_R(T) = -z(c + bT), \ 0 \le T \le L_1 \tag{12}$$

Solve the above equation with boundary condition I (L_1) = 0

$$I_R(T) = \frac{z}{\gamma^2}[e^{\gamma(L_1 - T)}(bL_1\gamma + c\gamma - b) - (\gamma bT + c\gamma - b)], \ 0 \le T \le L_1 \tag{13}$$

Maximum inventory level of the raw material, i.e. order quantity according to order from outer supplier

$$S_R = I_R(0) = \frac{z}{\gamma^2}\left[e^{L_1\gamma}(bL_1\gamma + c\gamma - b) - (c\gamma - b)\right] \tag{14}$$

Since the order is done at the time T = 0, thus the present worth of fuzzy ordering cost is

$$\widetilde{OoC}_R = \widetilde{a}_{3R} \tag{15}$$

The present worth of fuzzy holding cost for the raw material is

$$\widetilde{HoC}_R = \frac{\widetilde{a}_{1R}z}{\gamma^3}\left\{(e^{(\gamma L_1)} - 1)[bL_1\gamma + c\gamma - b] - \frac{\gamma L_1}{2}[bL_1\gamma + 2c\gamma - 2b]\right\} \tag{16}$$

The present worth of fuzzy deterioration cost for the raw material is

$$\widetilde{DoC}_R = \widetilde{a}_{2R}\frac{z}{\gamma^2}\left\{(e^{(\gamma L_1)} - 1)[bL_1\gamma + c\gamma - b] - \frac{\gamma L_1}{2}[bL_1\gamma + 2c\gamma - 2b]\right\} \tag{17}$$

Due to the cost of the item and the cost of the item being sold, the loss is incurred. Because the order is made on T = 0. The present value of the price of fuzzy items is

$$\widetilde{IoC}_R = \widetilde{a}_R Q_R = \widetilde{a}_R\frac{z}{\gamma^2}\left[e^{L_1\gamma}(bL_1\gamma + c\gamma - b) - (c\gamma - b)\right] \tag{18}$$

Therefore, the present worth of the total cost during the cycle is the cost of ordering costOoC_R, holding cost HoC_R, deterioration cost DoC_R and item cost IoC_R. Thus, for the raw materials, the present worth of total fuzzy cost is

$$\widetilde{ToC}_{1R} = \widetilde{OoC}_R + \widetilde{HoC}_R + \widetilde{DoC}_R + \widetilde{IoC}_R \tag{19}$$

Now, Present worth of total fuzzy profit is

$$\widetilde{ToP}_1 = \widetilde{SoR} - \widetilde{PoC} - \widetilde{HoC} - \widetilde{DoC} - \widetilde{OoC} - \widetilde{ToC}_{1R} \tag{20}$$

Observation 1. Total fuzzy profit ($\widetilde{TP}_{R1}$) is the function of L_1. Now we have to find the optimal value of L_1 in order to maximize the total profit $\widetilde{TP}_{R1}$ (L_1) subject to the inequality constraint S ≤ S_0, Now, we have LPP, **Max.** $\widetilde{TP}_{R1}$ **(L_1); Subject to $S_0 - S \ge 0$** \qquad (21)

Case B: $S \geq S_0$. In this case, the output starts with zero time, at time $T = 0$ and ends when the inventory level reaches level S, whereas $T = L_2$, where $S \geq S_0$. Initially the demand and the production rate are constant up to the time $T = L_1$ at which inventory level reaches the level S_0 after that demand becomes stock dependent so as the production rate until the time $T = L_2$, after that inventory level is reduced because of the combined outcome of the demand and the deterioration spreads the zero level at the time $T = L$. The inventory level of this model can be explained by the following differential equations and graphically represented in Figure 5.
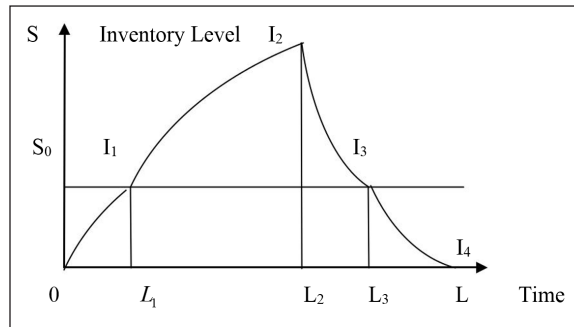

Figure 5. Graphical representation of case B: $S > S_0$

$$I_1'(T) + \gamma I_1(T) = (z-1)D, \ 0 \leq T \leq L_1 \tag{22}$$

$$I_2'(T) + \gamma I_2(T) = (z-1)(\alpha + \beta I_2(T)), \ L_1 \leq T \leq L_2 \tag{23}$$

$$I_3'(T) + \gamma I_3(T) = -[\alpha + \beta I_3(T)], \quad L_2 \leq T \leq L_3 \tag{24}$$

$$I_4'(T) + \gamma I_4(T) = -D, \ L_3 \leq T \leq L \tag{25}$$

Where the boundary limits are $I_1(0) = 0$, $I_2(L_1) = S_0$, $I_3(L_3) = S_0$, $I_4(L) = 0$.

Now solutions of above (22), (23), (24), (25) equations are:

$$I_1(T) \ = \frac{(z-1)D}{\gamma} - \frac{(z-1)D}{\gamma} e^{(-\gamma T)} = \frac{(z-1)D}{\gamma}[1 - e^{(-\gamma T)}], \ 0 \leq T \leq L_1 \tag{26}$$

$$I_2(T) = \frac{\alpha(z-1)}{(\gamma - z\beta + \beta)} + \left[S_0 - \frac{\alpha(z-1)}{(\gamma - z\beta + \beta)} \right] e^{(\gamma - z\beta + \beta)(L_1 - T)}, \ L_1 \leq T \leq L_2 \tag{27}$$

$$I_3(T) = -\frac{\alpha}{(\gamma + \beta)} + \left[S_0 + \frac{\alpha}{(\gamma + \beta)} \right] e^{(\gamma + \beta)(L_3 - T)}, \quad L_2 \leq T \leq L_3 \tag{28}$$

$$I_4(T) = \frac{De^{(\gamma L)}}{\gamma e^{(\gamma T)}} - \frac{D}{\gamma} = \frac{D}{\gamma} e^{\gamma(L-T)} - 1, \quad L_3 \le T \le L \tag{29}$$

Let $I_1 \; L_1 = S_0$, so equation (26) is

$$e^{\gamma L_1} = \frac{(z-1)D}{(z-1)D - S_0\gamma}, \quad L_1 = \frac{1}{\gamma} \log\left[\frac{(z-1)D}{(z-1)D - S_0\gamma}\right] \tag{30}$$

Now, equations from (27) and (28), Let $I_2 L_{2} = I_3 L_2$, So

$$L_3 = L_2 + \frac{1}{\beta+\gamma} \log\left\{\frac{1}{S_0 + \frac{\alpha}{\beta+\gamma}}\left[\frac{\alpha(z-1)}{\beta+\gamma-z\beta} + \left(S_0 - \frac{\alpha(z-1)}{\beta+\gamma-z\beta}\right)e^{(\beta+\gamma-z\beta)(L_1-L_2)} + \frac{\alpha}{\beta+\gamma}\right]\right\} \tag{31}$$

Now equation from (29) Put condition $I_4 L_3 = S_0$

$$L = L_3 + \frac{1}{\gamma} \log\left[\frac{S_0 * \gamma}{D} + 1\right] \tag{32}$$

Value of L_3 put in equation (32), so

$$L = L_2 + \frac{1}{\beta+\gamma} \log\left\{\frac{1}{S_0 + \frac{\alpha}{\beta+\gamma}}\left[\frac{\alpha(z-1)}{\beta+\gamma-z\beta} + \left(S_0 - \frac{\alpha(z-1)}{\beta+\gamma-z\beta}\right)e^{(\beta+\gamma-z\beta)(L_1-L_2)} + \frac{\alpha}{\beta+\gamma}\right]\right\} + \frac{1}{\gamma} \log\left[\frac{S_0 * \gamma}{D} + 1\right] \tag{33}$$

So, maximum inventory level is $S = I_2 (L_2)$

$$S = I_2(L_2) = \frac{\alpha(z-1)}{(\gamma - z\beta + \beta)} + \left[S_0 - \frac{\alpha(z-1)}{(\gamma - z\beta + \beta)}\right]e^{(\gamma - z\beta + \beta)(L_1 - L_2)} \tag{34}$$

Now, fuzzy holding cost is

$$\widetilde{HC} = \widetilde{\%_1}\left[\int_0^{L_1} I_1(T)dT + \int_{L_1}^{L_2} I_2(T)dT + \int_{L_2}^{L_3} I_3(T)dT + \int_{L_3}^{L} I_4(T)dT\right] \tag{35}$$

$$\widetilde{HC} = \widetilde{\%_1}\frac{D(z-1)}{\gamma}\left[L_1 - \frac{1}{\gamma} + \frac{e^{(-\gamma L_1)}}{\gamma}\right] +$$

$$\frac{\widetilde{\%_1}}{(\gamma+\beta-z\beta)}\left[\alpha L_1(z-1) + \alpha L_2(z-1) + S_0 - S_0 e^{(L_1-L_2)(\gamma+\beta-z\beta)} + \frac{\alpha(z-1)e^{(L_1-L_2)(\gamma+\beta-z\beta)}}{(\gamma+\beta-z\beta)} - \frac{\alpha(z-1)}{(\gamma+\beta-z\beta)}\right]$$

$$+ \frac{\widetilde{\%_1}}{(\gamma+\beta)}\left[\alpha L_2 - \alpha L_3 + S_0 e^{(L_3-L_2)(\gamma+\beta)} - S_0 + \frac{\alpha}{(\beta+\gamma)}S_0 e^{(L_3-L_2)(\gamma+\beta)} - \frac{\alpha}{\beta+\gamma}\right] + \widetilde{\%_1}\frac{D}{\gamma}\left[L_3 - \frac{1}{\gamma} - L + \frac{e^{\gamma(L-L_3)}}{\gamma}\right] \tag{36}$$

fuzzy deterioration cost is

$$\widetilde{D_0C} = \widetilde{\%_2}\gamma\left[\int_0^{L_1} I_1(T)dT + \int_{L_1}^{L_2} I_2(T)dT + \int_{L_2}^{L_3} I_3(T)dT + \int_{L_3}^{L} I_4(T)dT\right]$$

$$\widetilde{D_0C} = \widetilde{\%_2}D(z-1)\left[L_1 - \frac{1}{\gamma} + \frac{e^{(-\gamma L_1)}}{\gamma}\right] +$$

$$\frac{\widetilde{\%_2}\gamma}{(\gamma+\beta-z\beta)}\left[\alpha L_1(z-1) + \alpha L_2(z-1) + S_0 - S_0 e^{(L_1-L_2)(\gamma+\beta-z\beta)} + \frac{\alpha(z-1)e^{(L_1-L_2)(\gamma+\beta-z\beta)}}{(\gamma+\beta-z\beta)} - \frac{\alpha(z-1)}{(\gamma+\beta-z\beta)}\right]$$

$$+\frac{\widetilde{\%_2}\gamma}{(\gamma+\beta)}\left[\alpha L_2 - \alpha L_3 + S_0 e^{(L_3-L_2)(\gamma+\beta)} - S_0 + \frac{\alpha}{(\beta+\gamma)}S_0 e^{(L_3-L_2)(\gamma+\beta)} - \frac{\alpha}{\beta+\gamma}\right] + \widetilde{\%_2}D\left[L_3 - \frac{1}{\gamma} - L + \frac{e^{\gamma(L-L_3)}}{\gamma}\right]$$

$$(37)$$

fuzzy production cost is

$$\widetilde{P_0C} = \widetilde{p_2}D(z-1)\left[L_1 - \frac{1}{\gamma} + \frac{e^{(-\gamma L_1)}}{\gamma}\right] +$$

$$\frac{\widetilde{p_2}\gamma}{(\gamma+\beta-z\beta)}\left[\alpha L_1(z-1) + \alpha L_2(z-1) + S_0 - S_0 e^{(L_1-L_2)(\gamma+\beta-z\beta)} + \frac{\alpha(z-1)e^{(L_1-L_2)(\gamma+\beta-z\beta)}}{(\gamma+\beta-z\beta)} - \frac{\alpha(z-1)}{(\gamma+\beta-z\beta)}\right]$$

$$(38)$$

fuzzy sales revenue is

$$\widetilde{S_0R} = \frac{\widetilde{\%_2}D(z-1)}{\gamma}\left[L_1 - \frac{1}{\gamma} + \frac{e^{(-\gamma L_1)}}{\gamma}\right] +$$

$$\frac{\widetilde{\%_2}}{(\gamma+\beta-z\beta)}\left[\alpha L_1(z-1) + \alpha L_2(z-1) + S_0 - S_0 e^{(L_1-L_2)(\gamma+\beta-z\beta)} + \frac{\alpha(z-1)e^{(L_1-L_2)(\gamma+\beta-z\beta)}}{(\gamma+\beta-z\beta)} - \frac{\alpha(z-1)}{(\gamma+\beta-z\beta)}\right]$$

$$+\frac{\widetilde{\%_2}}{(\gamma+\beta)}\left[\alpha L_2 - \alpha L_3 + S_0 e^{(L_3-L_2)(\gamma+\beta)} - S_0 + \frac{\alpha}{(\beta+\gamma)}S_0 e^{(L_3-L_2)(\gamma+\beta)} - \frac{\alpha}{\beta+\gamma}\right] + \widetilde{\%_2}\frac{D}{\gamma}\left[L_3 - \frac{1}{\gamma} - L + \frac{e^{\gamma(L-L_3)}}{\gamma}\right]$$

$$(39)$$

fuzzy ordering cost is $\widetilde{O_0C} = \widetilde{\%_3}$ $\hspace{4cm}(40)$

Raw Material Inventory Model during the Time (0, L)

Originally, the vendor purchased the raw material in lots and then produces the finalized material. The Vendor starts production at time Zero (T = 0), and the raw material reaches the zero level at the time (T = L_2) because of the collective effect of production and

deterioration. The vendor's raw materials inventory system at any time T can be denoted by the following differential equation and has shown in Figure 6.
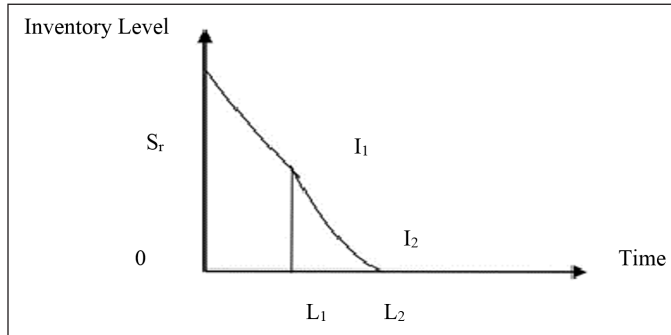

Figure 6. Graphical representation of raw material inventory

$$I_{R1}'(T) + \gamma I_{R1}(T) = -z(c + bT), \; 0 \le \text{T} \le L_1 \tag{41}$$

$$I_{R2}'(T) + \; I_{R2}(T) = -z[\alpha + \beta I_2(T)], \; L_1 \le \text{T} \le \text{L}_2 \tag{42}$$

The value of $I_2(T) = \dfrac{\alpha(z-1)}{(\gamma - z\beta + \beta)} + \left[S_0 - \dfrac{\alpha(z-1)}{(\gamma - z\beta + \beta)} \right] e^{(\gamma - z\beta + \beta)(L_1 - T)}$ from the

equation (27), So

$$I_{R2}'(T) + \gamma I_{R2}(T) = -z \left\{ \alpha + \beta \left\{ \frac{\alpha(z-1)}{(\gamma - z\beta + \beta)} + \left[S_0 - \frac{\alpha(z-1)}{(\gamma - z\beta + \beta)} \right] e^{(\gamma - z\beta + \beta)(L_1 - T)} \right\} \right\} \tag{43}$$

Where the boundary conditions are $I_{R1}(0) = S_R$ and $I_{R2}(L_2) = 0$,

Now Solution of above equations (41) and (43) are

$$I_{R1}(T) = -zT(c + bT) + S_R e^{(-T\gamma)}, \; 0 \le \text{T} \le L_1 \tag{44}$$

$$I_{R2}(T) = -\frac{z\alpha}{\gamma} - \frac{z(z-1)\beta\alpha}{\gamma(\gamma + \beta - z\beta)} - \frac{zS_0 e^{(\gamma + \beta - z\beta)(L_1 - T)}}{(z-1)} + \frac{z\alpha e^{(\gamma + \beta - z\beta)(L_1 - T)}}{(\gamma + \beta - z\beta)} + \frac{z\alpha e^{\gamma(L_2 - T)}}{\gamma}$$

$$\frac{z(z-1)\alpha\beta e^{\gamma(L_2 - T)}}{\gamma(\gamma + \beta - z\beta)} + \frac{zS_0 e^{(\gamma + \beta - z\beta)(L_1 - L_2) + (\gamma L_2)}}{(z-1)e^{T\gamma}} - \frac{z\alpha e^{(\gamma + \beta - z\beta)(L_1 - L_2) + (\gamma L_2) - (\gamma T)}}{(\gamma + \beta - z\beta)}, L_1 \le T \le L_2 \tag{45}$$

Now using the condition $I_{R1}(L_1) = I_{R2}(L_1)$, The order quantity per order from outside suppliers. The maximum inventory level of the raw materials.

$$S_R = zL_1(c + bL_1)e^{\gamma L_1} - \frac{z\alpha e^{\gamma L_1}}{\gamma} - \frac{e^{\gamma L_1}z(z-1)\beta\alpha}{z(\gamma+\beta-z\beta)} - \frac{ze^{\gamma L_1}S_0}{(z-1)} + \frac{e^{\gamma L_1}z\alpha}{\gamma+\beta-\beta z} +$$
$$\frac{z\alpha e^{\gamma L_2}}{\gamma} + \frac{z(z-1)\beta\alpha e^{\gamma L_2}}{\gamma(\gamma+\beta-z\beta)} + \frac{zS_0e^{\gamma L_2}e^{(L_1-L_2)(\beta+\gamma-z\beta)}}{(z-1)} - \frac{z\alpha e^{\gamma L_2}e^{(L_1-L_2)(\beta+\gamma-z\beta)}}{(\gamma+\beta-z\beta)}$$

$$(46)$$

From the order is done at the time (T= 0), then the fuzzy ordering cost is

$$\widetilde{OC} = \widetilde{S}_{3R} \tag{47}$$

Now, Fuzzy holding cost is $\widetilde{HC}_R = \widetilde{S}_{1R}\left[\int_0^{L_1} I_{R1}(T)dT + \int_{L_1}^{L_2} I_{R2}(T)dT\right]$

$$\widetilde{HC}_R = \widetilde{S}_{1R}\begin{bmatrix} S_R\left(\dfrac{1}{\gamma} - \dfrac{e^{-\gamma L_1}}{\gamma}\right) - \dfrac{z(2bL_1^3 + 3cL_1^2)}{6} - \dfrac{z\alpha L_2}{\gamma} + \dfrac{z\alpha L_1}{\gamma} + \dfrac{z(z-1)\beta\alpha L_1}{\gamma(\gamma+\beta-z\beta)} \\[3mm] -\dfrac{z(z-1)\beta\alpha L_2}{\gamma(\gamma+\beta-z\beta)} + \dfrac{zS_0e^{(L_1-L_2)(\gamma+\beta-z\beta)}}{(\gamma+\beta-z\beta)(z-1)} - \dfrac{zS_0}{(\gamma+\beta-z\beta)(z-1)} - \dfrac{z\alpha e^{(L_1-L_2)(\gamma+\beta-z\beta)}}{(\gamma+\beta-z\beta)^2} \\[3mm] +\dfrac{z\alpha}{(\gamma+\beta-z\beta)^2} + \dfrac{z\alpha e^{(L_2-L_1)}}{\gamma^2} - \dfrac{z\alpha}{\gamma^2} - \dfrac{z\beta\alpha(z-1)}{(\gamma+\beta-z\beta)\gamma^2} + \dfrac{\alpha\beta z(z-1)e^{\gamma(L_2-L_1)}}{(\gamma+\beta-z\beta)\gamma^2} \\[3mm] +\dfrac{zS_0e^{(-z\beta L_1+z\beta L_2-\beta L_2+\beta L_1)}}{\gamma(z-1)} - \dfrac{zS_0e^{(L_1-L_2)(\gamma+\beta-z\beta)}}{\gamma(z-1)} + \dfrac{z\alpha e^{(L_1-L_2)(\gamma+\beta-z\beta)}}{(\gamma+\beta-z\beta)\gamma} - \dfrac{z\alpha e^{\gamma(L_2-L_1)}e^{(L_1-L_2)(\gamma+\beta-z\beta)}}{(\gamma+\beta-z\beta)(\gamma)} \end{bmatrix}$$

$$(48)$$

So, Fuzzy deterioration cost is

$$\widetilde{DC}_R = \widetilde{S}_{2R}\gamma\left[\int_0^{L_1} I_{R1}(T)dT + \int_{L_1}^{L_2} I_{R2}(T)dT\right]$$

$$\widetilde{DC}_R = \widetilde{S}_{2R}\gamma\begin{bmatrix} S_R\left(\dfrac{1}{\gamma} - \dfrac{e^{-\gamma L_1}}{\gamma}\right) - \dfrac{z(2bL_1^3 + 3cL_1^2)}{6} - \dfrac{z\alpha L_2}{\gamma} + \dfrac{z\alpha L_1}{\gamma} + \dfrac{z(z-1)\beta\alpha L_1}{\gamma(\gamma+\beta-z\beta)} \\[3mm] -\dfrac{z(z-1)\beta\alpha L_2}{\gamma(\gamma+\beta-z\beta)} + \dfrac{zS_0e^{(L_1-L_2)(\gamma+\beta-z\beta)}}{(\gamma+\beta-z\beta)(z-1)} - \dfrac{zS_0}{(\gamma+\beta-z\beta)(z-1)} - \dfrac{z\alpha e^{(L_1-L_2)(\gamma+\beta-z\beta)}}{(\gamma+\beta-z\beta)^2} \\[3mm] +\dfrac{z\alpha}{(\gamma+\beta-z\beta)^2} + \dfrac{z\alpha e^{(L_2-L_1)}}{\gamma^2} - \dfrac{z\alpha}{\gamma^2} - \dfrac{z\beta\alpha(z-1)}{(\gamma+\beta-z\beta)\gamma^2} + \dfrac{\alpha\beta z(z-1)e^{\gamma(L_2-L_1)}}{(\gamma+\beta-z\beta)\gamma^2} \\[3mm] +\dfrac{zS_0e^{(-z\beta L_1+z\beta L_2-\beta L_2+\beta L_1)}}{\gamma(z-1)} - \dfrac{zS_0e^{(L_1-L_2)(\gamma+\beta-z\beta)}}{\gamma(z-1)} + \dfrac{z\alpha e^{(L_1-L_2)(\gamma+\beta-z\beta)}}{(\gamma+\beta-z\beta)\gamma} - \dfrac{z\alpha e^{\gamma(L_2-L_1)}e^{(L_1-L_2)(\gamma+\beta-z\beta)}}{(\gamma+\beta-z\beta)(\gamma)} \end{bmatrix}$$

$$(49)$$

Now the loss of new item costs and the cost of the item sold is also included. Because the order is done at T = 0. And the cost of fuzzy items

$$\widetilde{IoC}_R = \widetilde{\alpha}_R S_R \tag{50}$$

So, the total cost during the cycle is the sum of ordering cost OoC_R, holding cost HoC_R, deterioration cost DoC_R and fuzzy item cost IoC_R. Thus, for the raw materials, the present worth of total fuzzy cost is

$$(\widetilde{TP})_{2R} = \widetilde{OoC}_R + \widetilde{HoC}_R + \widetilde{DoC}_R + \widetilde{IoC}_R \tag{51}$$

Now our the Present worth of total fuzzy profit is

$$\widetilde{TP}_{R2} = \widetilde{SoR} - \widetilde{PoC} - \widetilde{HoC} - \widetilde{DoC} - \widetilde{OoC} - \widetilde{IoC}_{2R} \tag{52}$$

Observation 2. The total fuzzy profit ($\widetilde{TP}_{R2}$) is the function of L_2. Now we have to find the optimum value of L_2 to maximize the total profit $\widetilde{TP}_{R2}$ (L_2) subject to the inequality constraint $S > S_0$, Now, we have LPP, **Max. $\widetilde{TP}_{R2}$ (L_2); Subject to S - S_0 > 0** (53)

Numerical Example

Case A: $S_0 \geq S$. The following values are used to interpret the model.

z = 1.2, b = 1.4, c=1.5, α=25, β=0, S_0 = 150, $\widetilde{\alpha}_1$= 0.45, 0.50, 0.60, $\widetilde{\alpha}_2$= 0.35, 0.40, 0.50, $\widetilde{\alpha}_3$ =130, 200, 300, $\widetilde{\alpha}_{R1}$ = 0.45, 0.50, 0.60, $\widetilde{\alpha}_{R2}$ = 0.35, 0.40, 0.50, $\widetilde{\alpha}_{3R}$ = 80, 100, 125, $\widetilde{\alpha}_R$ = 0.8, 1.0, 1.2

$\widetilde{p}_c$ = 4,5,7, $\widetilde{s}$ = 10, 12, 15, D = 0.1, I = 0.05, γ =0.01.

From the above parametric values, we get the results with the help of GA and the results are shown in Table 2.

Table 2
Results of optimal values

L_1	L	S_r	S	TP_{r1} (L_1)
4.14	4.95	47.93	5.84	960.46

We receive the optimum cyclic length L= 4.95, and the maximum inventory level S = 5.84 units and the output time is L_1= 4.14, the optimum gain is 960.46.

At the beginning of each cycle, the raw material of S_R = 47.93 units for the L_1 Purchases producer of P and produces the final product. In this time period, the raw material list decreases and on time T = 4.14, it will reach zero level, the list of final products will be build up time T = 4.14 and the inventory level will reach 5.84 units. At this time, production and

inventory ends due to the impact of manufacturer demand and deterioration. The inventory level reaches zero at the time level T = 4.95, then the output will start for the next cycle.

Sensitivity analysis. Effect of different parameters on Total Profit.

Table 3
Present value of total profits of the model due to z

S. No.	z	TP$_{r1}$
1.	1.3	969.24
2.	1.4	972.45
3.	1.5	958.41
4.	1.6	942.25

Table 4
Present value of total profits of the model due to α

S. No.	α	TP$_{r1}$
1.	20	756.12
2.	26	975.15
3.	27	985.48
4.	29	994.15

Table 5
Present value of total profits of the model due to β

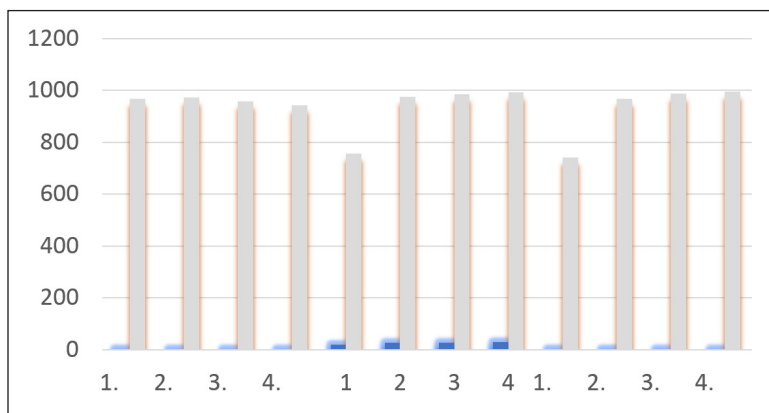| S. No.	β	TP$_{r1}$
1.	0.23	742.15
2.	0.27	969.49
3.	0.29	987.81
4.	0.30	996.17

Figure 7. Graphical representation of different parameters z, α, β and their Total Profit TP$_{r1}$

Observations.

1. From the above table number 3, 4, and 5 with different parameters, we look at the different values of total profit in relation to different production rates.
2. We have seen that if the production rate increases, then the total profit also gets the maximum increase in production but the profit will be reduced.
3. Due to the linear demand rate stock level is increasing and the production rate also increases.
4. If the production rate increases by fixed limit then decrease in optimum profit due to the increase in the cost of the fall.
5. The results of different values of demand parameters are obtained in table 4.
6. We have demanded in reality that the optimum profit increases if demand criteria are increasing.
7. With the effect of inflation and discount rate, the results of parametric values are present in Table 5.
8. We have seen that if the effect of inflation and rebate rate is on the rise then optimal profit is reduced.

Case B: $S_0 < S$. We are using the following values to describe the model.

z = 1.2, b = 1.4, c =1.5, α=25, β=0.25, S_0 = 150, $\%_c$= 0.45, 0.50, 0.60, $\%_2$= 0.35, 0.40, 0.50,

$\%_3$=130, 200, 300, $\%_{R1}$ = 0.45, 0.50, 0.60, $\%_{R2}$ = 0.35,0.40, 0.50, $\%_{3R}$ = 80, 100, 125, $\%_R$ = 0.8,1.0,1.2

$\%_c$ = 4,5,7, $\%$= 10, 12, 15, D = 0.1, I = 0.05, γ =0.01, R = 0.05.

With the above parametric values, we get results with the help of GA and the results are in Table 6.

We receive optimum cyclic length L= 87.14, and maximum inventory level S = 232.5 units and the production time L_1= 46.20, the optimum profit is 62348.154.

At the beginning of each cycle, S_r=847.481 units of the raw materials for the period L_1 purchases, a manufacturer of rate P and production the final product. In this time period, the raw material list decreases and at time T = 52.67, it will reach zero level, the inventory of final product will be T = 52.67 and the inventory level will reach 232.5 units. At this time, production and inventory ends due to the impact of manufacturer demand and fall. Inventory level reaches zero level at time T = 87.14, then production will start for next cycle.

Table 6
Results of optimal values

L_1	L_2	L_3	L	S_r	S	T P_{r2} (L_2)
46.20	52.67	45.27	87.14	847.481	232.5	62348.154

Sensitivity analysis. Effect of various parameters on Total Profit.

Table 7
Present value of total profits of the model due to z

S. No.	z	TP_{r2}
1.	1.25	94454.15
2.	1.30	135785.54
3.	1.35	945175.29
4.	1.40	1245475.595

Table 8
Present value of total profits of the model due to α

S. No.	α	TP_{r2}
1.	24	48972.28
2.	26	69547.53
3.	27	84981.34
4.	28	94724.87

Table 9
Present value of total profits of the model due to β

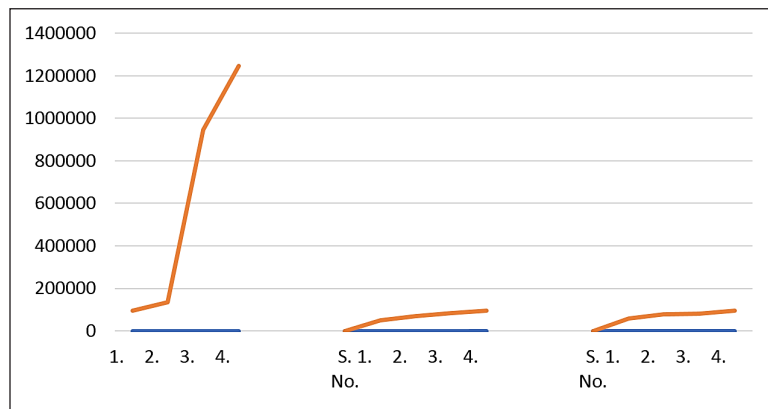| S. No.	β	TP_{r2}
	0.24	57415.26
	0.26	78445.58
	0.27	81156.24
	0.28	94584.15

Figure 8. Graphical representation of different parameters z, α, β and their Total Profit TP_{r2}

Observations.

1. From the above table number 7, 8, and 9 with different parameters, we look at the different values of total profit in relation to different production rates.
2. We have seen that if the production rate increases, then the total profit also gets the maximum increase in production but the profit will be reduced.
3. Due to the linear demand rate stock level is increasing and the production rate also increases.
4. If the production rate exceeds a certain limit, then the reduction in optimum profit due to increase in the cost of the cost of the holding and the cost of the fall.
5. Results in the table 8 are obtained for different standards of demand parameters.
6. We have demanded in reality that the optimum profit increases if demand criteria are increasing.
7. With the effect of inflation and discount rate, the results of parametric values are present in Table 9.
8. We have seen that if inflation and rebate rate are increasing, then optimum profit is also decreasing.

CONCLUSIONS AND FUTURE WORK

A production inventory model developed for newly launched product with the effect of inflation under fuzzy environment. And the demand rate is considered linearly stock dependent and shortage are not allowed. Model is formulated to maximize the expected profit from the full planning horizon. Also, we used to Genetic Algorithm (GA) for optimal extreme solutions for growth of business. A Genetic Algorithm with varying population size is used to solve the model where crossover probability is a function of parent's age type and it is obtained by fuzzy rule-base and possibility theory. And next section, the model is developed under the effect of inflation in fuzzy environment. This GA can also help in resolving the decision-making problems in the areas of science and technology and forecasting.

This fuzzy inventory model can be extended to supermarket price dependent demand, GDP policy, to storage and cold storage industry.

In this study, we develop an EPQ model with stock depended demand rate under the influence of inflation in a fuzzy environment, for the defuzzification we used centroid method and Genetic Algorithm (GA) is used for optimization.

So far, no researchers have considered these factors in their studies.

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